## Gamma-ray feeding and decay of superdeformed states

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**Abstract.** We report on four recent results concerning the population and the decay of superdeformed states, namely the structure of excited superdeformed states in  $^{194}$ Hg, the search for fine structure of the last superdeformed transitions in  $^{194}$ Pb, the primary decay-out strength analysis in  $^{194}$ Hg and, as a consequence of this, the possibility of using the decay-out as a tool to study order-to-chaos properties of normally deformed states.

**PACS.** 21.10.Re Collective levels – 23.20.Lv  $\gamma$  transitions and level energies – 24.60.-k Statistical theory and fluctuations – 24.60.Lz Chaos in nuclear systems – 27.80.+w 190  $\leq A \leq 219$ 

### 1 Excited superdeformed states

Despite intense experimental efforts, not much is known about the collective  $\gamma$ -ray cascades populating superdeformed (SD) states. The nucleus which has been most extensively studied in this respect is <sup>143</sup>Eu. In this nucleus, two different regimes have been identified [1]: a very intense E2 bump (~ 50% of the reaction chanel) which is interpreted as being due to damped rotational transitions and an intense ridge structure (~ 3%) which corresponds to the two-dimensional correlation pattern of unresolved transitions which form discrete excited bands. Fluctuation analysis techniques have been applied to the ridge structures and it has been found that  $\sim$  20–30 bands are sampled in this stage of the cascade. The ratio of second to first ridge intensity, which corresponds to the inband probability, is about 40% [2,3]. The number of these bands decreases at low spin and this is interpreted as being due to the decay-out of these bands through mixing with normally deformed (ND) states [4]. A very remarkable feature is that the ridges do not seem to populate the yrast SD band, since no ridges are observed when gating on SD yrast transitions.

A recent analysis of thin target GAMMASPHERE <sup>194</sup>Hg data has revealed very different results: The spectrum of gamma-rays emitted in coincidence with the yrast SD band in <sup>194</sup>Hg, for which excitation energies, spins and parities are well established [5], shows a broad bump extending from 400 keV to 1 MeV. This bump is characteristic of this mass region and has always been associated with the collective transitions feeding into the yrast SD

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Fig. 1. Double SD-gated  $E_{\gamma}$ - $E_{\gamma}$  (in keV) coincidence matrix in <sup>194</sup>Hg. Compton and other uncorrelated events have been subtracted using the COR procedure [6]. The yrast SD band shows up as a regular grid pattern with an empty diagonal. The first and second ridge are clearly visible from 650 to 850 keV.

line since it peaks at a rotational frequency corresponding to 50% feeding of the yrast band. In a two-dimensional SDgated spectrum (see fig. 1), two (and maybe three) ridges are clearly visible from 500 to 900 keV. This is the first time that  $E_{\gamma}$ - $E_{\gamma}$  ridges are observed in coincidence with the yrast SD lines. There is a one-to-one correspondence between the ridges and the 1-dimensional E2 bump up to 800 keV and, beyond this energy, the 1D bump never exceeds twice the ridge intensity: this means that rotational correlations are very strong in this nucleus, as compared to  $^{143}$ Eu, where the bump is more than ten times stronger than the ridges. Also, it turns out that the inband probability of these ridges is  $\sim 1$  until feeding into the yeast SD band is forced because of the gating condition. Perhaps the most interesting feature is that the ridges are much narrower (FWHM  $\sim 8$  keV) than the ridges seen in normally deformed nuclei (FWHM  $\sim 30$  keV) and also in SD  $^{143}$ Eu [4] and that their width increases with transition energy. This increase in the width denotes a more fragmented rotational E2 strength at higher spins where the E2 cascades go probably through hotter states.

A preliminary fluctuation analysis performed on the first ridge in  $^{194}$ Hg has shown that there are many more excited bands (~ 100–150) in this nucleus than in  $^{143}$ Eu. This feature had been predicted by Yoshida and Matsuo [7] and is due to the very small rotational damping width in this mass region (of the order of the level spacing in the energy interval of interest, 1.2–2.0 MeV). Since the E2 strength is not fragmented over many final states, rotational structures can survive to higher excitation energy, and therefore there are effectively more of them. A

more detailed inspection of the calculations shows that these bands have many components of intrinsic states in their wave function (5 on average but up to ~ 10 at lower spins). The narrow widths of the ridges imply that the wave functions, including the relative phases, do not vary much as the spin increases by  $2\hbar$ . Phase and rotational coherence seems to be maintained. These excited bands are therefore strongly interacting and yet "extremely rotationally coherent" bands (maybe even the ergodic bands predicted by Mottelson [8]). An experiment to investigate this subject in the <sup>192</sup>Hg nucleus with more statistics has just been carried out at EUROBALL IV.

#### 2 Fine structure of superdeformed transitions

The decay from SD states occurs because of mixing with ND states: the SD wave function acquires an amplitude  $\alpha^2$  at normal deformation. The SD state has a partial width  $(1 - \alpha^2) \Gamma_{SD}$  to decay to the next SD state and a partial width  $\alpha^2 \Gamma_{\rm ND}$  to decay to lower-energy ND states. If the coupling between ND and SD states is weak, the mixing will result in two states which are populated from the previous SD state proportionaly to their SD amplitude squared:  $(1 - \alpha^2)$  for the predominently SD state and  $\alpha^2$ for the predominantly ND state. SD transitions populating SD decaying states should then effectively be doublets. This is clearly observed in the mass 130 [9] and 160 [10] regions. In the mass 190 region, however, the excitation energy of SD states is such that the average spacing between SD states and their closest ND neighbours is on average very small (tens of eV). This is too small to resolve, except maybe in the <sup>194</sup>Pb case, where the excitation energy of SD states at the point of decay is lower and where the density of ND states is reduced because <sup>194</sup>Pb is a semi-magic nucleus at normal deformation. We have performed two experiments, one with Yrasball at Yale and the other with GASP in Legnaro, replacing in both cases five large-volume germanium detectors by LEPS detectors (Low Energy Photon Spectrometers). This is because their good energy resolution at low energy (500–700 eV at 121 keV in LEPS detectors as compared to 1.0–1.2 keV in large-volume detectors) was essential if any splitting of the last SD transitions at 169 and 213 keV was to be observed. An analysis of the GASP data has revealed some strength to the left of the 169 keV transition in the spectrum of  $\gamma$ -rays detected in the LEPS detectors in coincidence with the SD band (panel a) of fig. 2). This excess strength is also visible in the spectrum of  $\gamma$ -rays detected in the large-volume detectors (panel b) in fig. 2): it seems indeed that the 169 keV transition has a shoulder towards low energies. The ratio of the intensities of the main SD  $8^+-6^+$  peak and of its satellite yields a 15(5)% ND admixture into the  $6^+$  SD state. The energy separation between the two peaks is 1.3(2) keV after interaction and 0.9(2)before interaction. The interaction strength is extracted to be 0.6(2) keV. The evidence for splitting is present but weak and further statistics needs to be collected.



Fig. 2. a) SD-gated spectrum of  $\gamma$ -rays detected in the LEPS detectors. b) SD-gated spectrum detected in the large-volume detectors which had the best experimental resolution (7 out of 35). The SD symbols denote SD lines, the Y's denote ND yrast lines. The insets correspond to zooms around the 169 keV SD transition and the dashed line is to guide the eye.

### 3 Primary strength distribution

One of the questions that is often asked about the decay from SD states is whether the decay is a statistical process and whether fluctuations are important. Since it is the ND component of the SD state wave function which governs the decay-out, asking this question is equivalent to asking whether the ND states to which the SD states couple through the potential barrier are compound states or not. There are many experimental observations which support this: the large excitation energy of SD states at the point of decay [5,11–13], the quasi-continuous nature of the decay spectrum [14,15], the large fragmentation of the decay cascades [16], the fact that statistical decay calculations reproduce the general shape of the decay spectrum [17] and the similarity between the decay spectrum and the spectrum of  $\gamma$ -rays following resonant neutron capture at 7-8 MeV above yrast [18]. In the neutron resonance case, the statistical nature of the decay is well established [19]: the strengths of the primary transitions follow a  $\chi^2$  distribution with  $\nu = 1$  degree of freedom, often called a Porter-Thomas distribution [20]. This is a direct consequence of random matrix theory, and, in particular, it reflects the properties of the Gaussian Orthogonal Ensemble (GOE) of random matrices [21]. The particularity of the Porter-Thomas distribution is that, unlike other  $\chi^2$  distributions with higher degrees of freedom, it diverges towards low strengths, yet it also extends to very large strengths. This property gives rise to very strong intensity fluctuations which could be the reason why single-step decay transitions are sometimes enhanced and can be observed experimentally. This is the case in <sup>194</sup>Hg, for which strong highenergy links have been identified while in the neighbouring <sup>192</sup>Hg, studied with similar statistics, no such lines



Fig. 3. On the top: a) experimental decay spectrum in  $^{194}$ Hg, b) and c) simulated spectra obtained from the same 600 primary lines (represented as impulses) but with 2 different random number generator seeds for producing the counting statistics. On the bottom: decay spectra in  $^{192}$ Hg obtained at GAM-MASPHERE (top spectrum) and EUROGAM 2 (bottom spectrum). The strongest peaks (marked by an arrow) survive the different experiments (or random generator seeds) although they are slightly altered but the weaker lines can sometimes disappear.

could be observed. In order to show that the enhancement of the strengths in <sup>194</sup>Hg may be due to Porter-Thomas fluctuations, a study of the primary strength distribution was performed. The aim was to determine which  $\chi^2$  distribution of  $\nu$  degrees of freedom and average strength  $\theta$  could best fit the experimental strengths  $\omega_i$  extracted above the experimental strength threshold  $\omega_{\text{low}}$ . The result for the most likely  $\chi^2$  distribution is the following:  $\nu = 1$  and  $\theta$  is found to be nearly four times smaller than the experimental strength threshold [22]. The uncertainty on the number of degrees of freedom  $\nu$  is very large because only the high-strength tail of the distribution is accessible experimentally and this is a strength domain for which there is not a pronounced difference between  $\chi^2$  distributions. In other words, only the strongest 19 strengths are observed, whereas a fluctuation analysis in the same transition energy interval  $(E_{\gamma} > 2.6 \text{ MeV})$  tells us there should be ~ 600 [16]. Simulations of the decay spectrum were performed: 600 strengths were sampled from the most likely distribution and their associated energies were randomly chosen from an inverse level-density formula. The experimental resolution was then folded in as well as the Compton background, the underlying statistical feeding spectrum and counting statitistics. The simulations are very representative of the data in the sense that the simulated spectra look very much like the experimental spectra. They also can account for the fact that, in some cases, strong lines are present at high energy (as in  $^{194}$ Hg) and in some cases, they are not (as in  $^{192}$ Hg).

Two important observations came out of the simulations and these are illustrated in fig. 3. Firstly, the fact that a peak in the simulations stems very rarely from one line. This is specially true towards low energy since the inverse level-density energy distribution yields many more lines at lower energies than at higher ones. The lines pile up one on top of each other and yield very broad and/or funny-shaped lines. This is what is called the *pandemo*nium effect. Secondly, the effect of the counting statistics is not negligible. Given a set of simulated lines, if one changes the seed of the random number generator in order to produce different counting statistics, the shape and the intensity of the lines vary dramatically: some lines which are present and sharp in one spectrum disappear or become wide or even doublets in the other. Experimentally, this seed change is equivalent to doing the experiment a second time. This tells us that caution is needed when treating peaks at the very limit of the resolving power of multidetector arrays and that setting a  $3\sigma$  limit on the intensity of lines is the absolute minimum criterium to define a peak.

# 4 Order-to-chaos properties of normally deformed states

The "chaoticity" of ND states has been addressed by a number of people in different regions of the (E, I)-plane (see fig. 4). The neutron resonances have been investigated [23], as well as near yrast levels in different nuclei [24]. A complete study of levels has been performed in  ${}^{116}$ Sn [25] and  ${}^{26}$ Al [26]. All these studies were done on the basis of level spacing statistics. The decay from SD states provides new regions of the (E, I)-plane to study the chaoticity of ND states: at low spin in the fission isomers, at moderate spins in the mass 190 region and at higher spins in the mass 150 region. The bridge between the ordered and chaotic decay-out regimes can be made by introducing a chaoticity parameter [27]. In this picture, the SD state couples weakly only to specific ND states through the barrier. These states are called doorway states and they are equivalent to the collective HF+BCS states



Fig. 4. Regions of the (E, I)-plane where studies of orderto-chaos properties of ND states have been or will be (in the cases of mass 150 SD decay-out and the decay-back of fission isomers) performed.

that the GCM procedure mixes with the SD state [28–30]. Compound energy eigenstates of the ND spectrum are described by coupling the basis of  $|\mu\rangle$  states with a Hamiltonian matrix selected from the Gaussian Orthogonal Ensemble. However, if part of the structure of the basis states is kept in the energy eigenstates, the off-diagonal matrix elements are reduced, and this is performed by scaling them with a common chaoticity parameter  $\Delta$  ( $0 \leq \Delta \leq 1$ ). The matrix is then diagonalised. After this procedure, the SD state  $|sd\rangle$  is included in the middle of the spectrum together with its weak coupling V to the doorway state  $|d\rangle$  and one more diagonalisation is carried out. If  $\Delta = 0$ , the ND states do not mix and the SD state can only acquire an admixture of the nearest collective doorway state. When  $\Delta$  is equal to 1, the ND spectrum is fully mixed and the doorway state has dissolved among all the ND states. Since the coupling V is weak, one of the final states  $|S\rangle$ , which has a predominant SD component, will include many small admixtures of the original ND basis states  $|\mu\rangle$ . To proceed further, the conjecture is that admixtures can be viewed as strengths: each basis state  $|\mu\rangle$  is connected to one final state  $|\mu'\rangle$  at lower energy.  $N_{\rm s}$  simulations are carried out. For each simulation,  $N_{\rm t}$  strengths are chosen. This corresponds to selecting a specific interval of transition energies, usually associated to the highest energies. Out of these  $N_{\rm t}$ , the  $N_{\rm o}$  strongest (o for observed as in the experiment) are selected. The smallest of these  $N_{\rm o}$ strength is then equivalent to the experimental strength threshold and all the strengths are normalised to it. For a given set of  $\Delta$  and  $N_{\rm t}$ , 500 different GOE matrices of size N = 400 are diagonalised. The observational threshold is set to  $N_0 = 19$ , as in the <sup>194</sup>Hg case. The average cumulative distribution is computed: the number of strengths observed as a function of strength averaged over the 500 simulations. The cumulative comparison is then defined as the fraction of simulations which display a larger  $\chi^2$  deviation from the average cumulative distribution than the data. This is plotted in fig. 5 as a function of  $\Delta$  and  $N_{\rm t}$ . A data



Fig. 5. Perspective plot of the comparison to simulated cumulative functions for the 19 visible decay-out transitions in  $^{194}$ Hg.

set is considered to be successful if it compares better than 25% of the simulations. This condition is fulfilled when the total number of primary strengths is larger than 200 and when  $\Delta$  is larger than 0.1. This determination of the relevant parameters is in agreement with the measurement of 600 primary lines in the 2.6–5 MeV transition energy range and with the result of the most likely  $\chi^2$  distribution.

It therefore appears highly probable that the decay-out in  $^{194}{\rm Hg}$  is a statistical process and that the ND states to which the SD states couple at 4.2 MeV above yrast are compound states.

#### 5 Conclusion

The study of excited SD states in <sup>194</sup>Hg has revealed an example of extreme robustness of rotational correlations with excitation energy and spin. The detailed dependence of the E2 rotational strength on spin will be adressed in the near future with a large EUROBALL IV  $^{192}$ Hg data set. An experiment at GASP using five LEPS detectors has given evidence for the splitting of the last SD transition emitted by the <sup>194</sup>Pb nucleus. From this observation, the ND amplitude of the decaying SD  $6^+$  state as well as the coupling strength between SD and ND states at this spin could be extracted in a model-independent way. The analysis of the primary decay-out strengths in  $^{194}\mathrm{Hg}$ has shown that the decay from SD states in this nucleus is most probably a statistical process. A new method has been developed to study the strength distribution of primary  $\gamma$ -rays in any type of regime, from the ordered extreme to chaotic one, by introducing a chaoticity parameter. The decay-out of SD states offers in this way new regions of excitation and spin to study the order-to-chaos properties of ND states. Finally, the difficulties involved in identifying primary  $\gamma$ -rays and the problem of the "realness" of a peak was discussed. These experimental

limitations should soon be overcome by the advent of a new generation of  $\gamma$ -detector arrays such as AGATA [31] and GRETA [32], based on the reconstruction of photon trajectories in the germanium detectors.

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